工業反応装置特論

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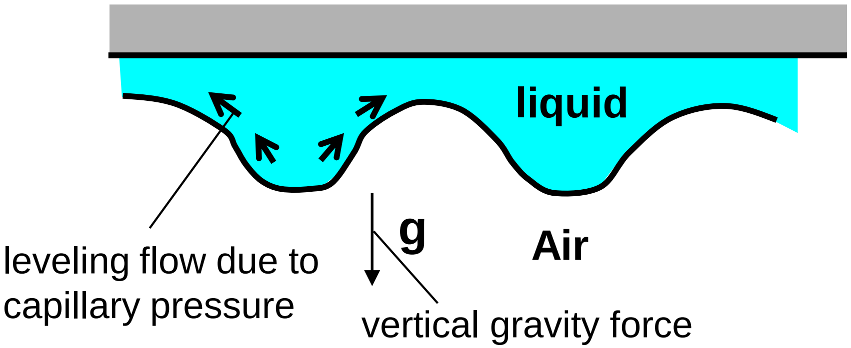
PERIODIC SURFACE DEFORMATION

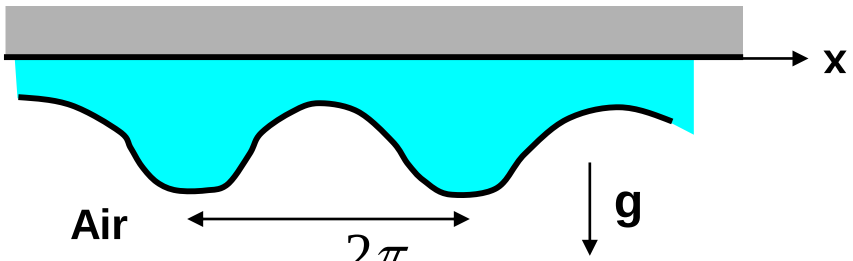


重力と表面張力のバランス ➡ 周期的な表面形状

FPEは次式で表される

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{h^3}{3\mu}\frac{d}{dx}\left(\sigma\frac{\partial^2 h}{\partial x^2} + \rho g h\right)\right] \qquad (1)$$

LINEAR STABILITY ANALYSIS (1)



$$\lambda (\equiv \frac{2\pi}{k})$$

線形安定性解析

・振幅ε_0の微小ゆらぎを考え、ゆらぎの成長、減衰を式(2)中の記号sの符号で判別

$$h = \quad \bar{h} \quad + \quad \varepsilon(t, x)$$

平均値　　変動成分

$$\varepsilon(t, x) = \varepsilon_0 \exp(st) \sin(\frac{x}{\lambda}) \qquad (2)$$

波長λの理想的sin波

s>0なら凹凸発達（不安定）　s<0なら安定　s=0は中立条件

(例）$s = 0.1, \quad \varepsilon_0 = 1\mathrm{nm}$（分子熱ゆらぎ), $\quad x = \lambda$のとき

$t = 100s$後には

$$\varepsilon(t, x) = (1 \times 10^{-9}) \exp(10) \sin(1)$$

$$= 2.2 \times 10^{-5} m = 22 \mu m$$

LINEAR STABILITY ANALYSIS (2)

粘度μ、表面張力σ、密度ρが座標xや時間によらず一定なら(1)から

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{h^3}{3\mu}\frac{d}{dx}\left(\sigma\frac{d^2h}{dx^2}+\rho gh\right)\right]$$

$$= -\frac{1}{3\mu}\frac{\partial}{\partial x}\left[\sigma h^3\left(\frac{d^3h}{dx^3}\right)+\rho gh^3\frac{dh}{dx}\right] \quad (3)$$

(2)を式(3)に代入することを考える。

まず時間微分は

$$\frac{\partial h}{\partial t} = \varepsilon_0 s\exp(st)\sin(\frac{x}{\lambda})$$

空間微分は

$$\frac{\partial h}{\partial x} = \varepsilon_0\frac{1}{\lambda}\exp(st)\cos(\frac{x}{\lambda})$$

$$\frac{\partial^2 h}{\partial x^2} = -\varepsilon_0\frac{1}{\lambda^2}\exp(st)\sin(\frac{x}{\lambda})$$

$$\frac{\partial^3 h}{\partial x^3} = -\varepsilon_0\frac{1}{\lambda^3}\exp(st)\cos(\frac{x}{\lambda})$$

LINEAR STABILITY ANALYSIS (3)

ここでε_0の2乗以上の項は非常に小さいので無視すると（線形近似）

$$h^3 = \left(\bar{h} + \varepsilon_0 \exp(st) \sin(\frac{x}{\lambda}) \right)^3$$

$$\sim \bar{h}^3 + 3\bar{h}\varepsilon_0 \exp(st) \sin(\frac{x}{\lambda})$$

従って(3)の右辺括弧内は

$$\sigma h^3 \frac{d}{dx}\left(\frac{d^2 h}{dx^2} \right) + \rho g h^3 \frac{dh}{dx}$$

$$= \sigma \left\{ \bar{h}^3 + 3\bar{h}\varepsilon_0 \exp(st) \sin(\frac{x}{\lambda}) \right\} \left\{ -\varepsilon_0 \frac{1}{\lambda^3} \exp(st) \cos(\frac{x}{\lambda}) \right\}$$

$$+ \rho g \left\{ \bar{h}^3 + 3\bar{h}\varepsilon_0 \exp(st) \sin(\frac{x}{\lambda}) \right\} \left\{ \varepsilon_0 \frac{1}{\lambda} \exp(st) \cos(\frac{x}{\lambda}) \right\}$$

$$= \sigma \bar{h}^3 \left\{ -\varepsilon_0 \frac{1}{\lambda^3} \exp(st) \cos(\frac{x}{\lambda}) \right\} \quad (\because \varepsilon_0{}^2 \approx 0)$$

$$+ \rho g \bar{h}^3 \left\{ \varepsilon_0 \frac{1}{\lambda} \exp(st) \cos(\frac{x}{\lambda}) \right\}$$

$$= \bar{h}^3 \left(-\frac{\sigma}{\lambda^2} + \rho g \right) \frac{1}{\lambda} \varepsilon_0 \exp(st) \cos(\frac{x}{\lambda})$$

LINEAR STABILITY ANALYSIS (4)

(3)に代入すると

$$\varepsilon_0 s \exp(st)\sin(\frac{x}{\lambda}) = -\frac{1}{3\mu}\frac{\partial}{\partial x}\left[\bar{h}^3\left(-\frac{\sigma}{\lambda^2}+\rho g\right)\frac{1}{\lambda}\varepsilon_0 \exp(st)\cos(\frac{x}{\lambda})\right]$$

$$= -\frac{1}{3\mu}\bar{h}^3\left(-\frac{\sigma}{\lambda^2}+\rho g\right)\frac{1}{\lambda}\varepsilon_0 \exp(st)\frac{\partial}{\partial x}\left[\cos(\frac{x}{\lambda})\right]$$

$$= -\frac{1}{3\mu}\bar{h}^3\left(-\frac{\sigma}{\lambda^2}+\rho g\right)\frac{1}{\lambda}\varepsilon_0 \exp(st)\cdot\frac{1}{\lambda}\left(-\sin(\frac{x}{\lambda})\right)$$

整理すれば

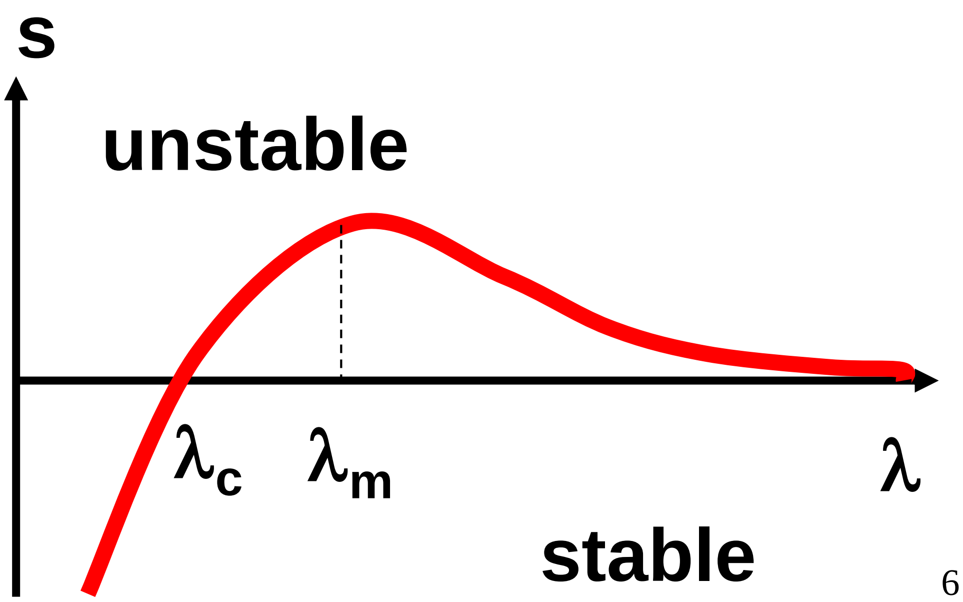
$$s = \frac{1}{3\mu}\bar{h}^3\left(-\frac{\sigma}{\lambda^2}+\rho g\right)\frac{1}{\lambda^2} \quad (4)$$

安定である条件は$s<0$から

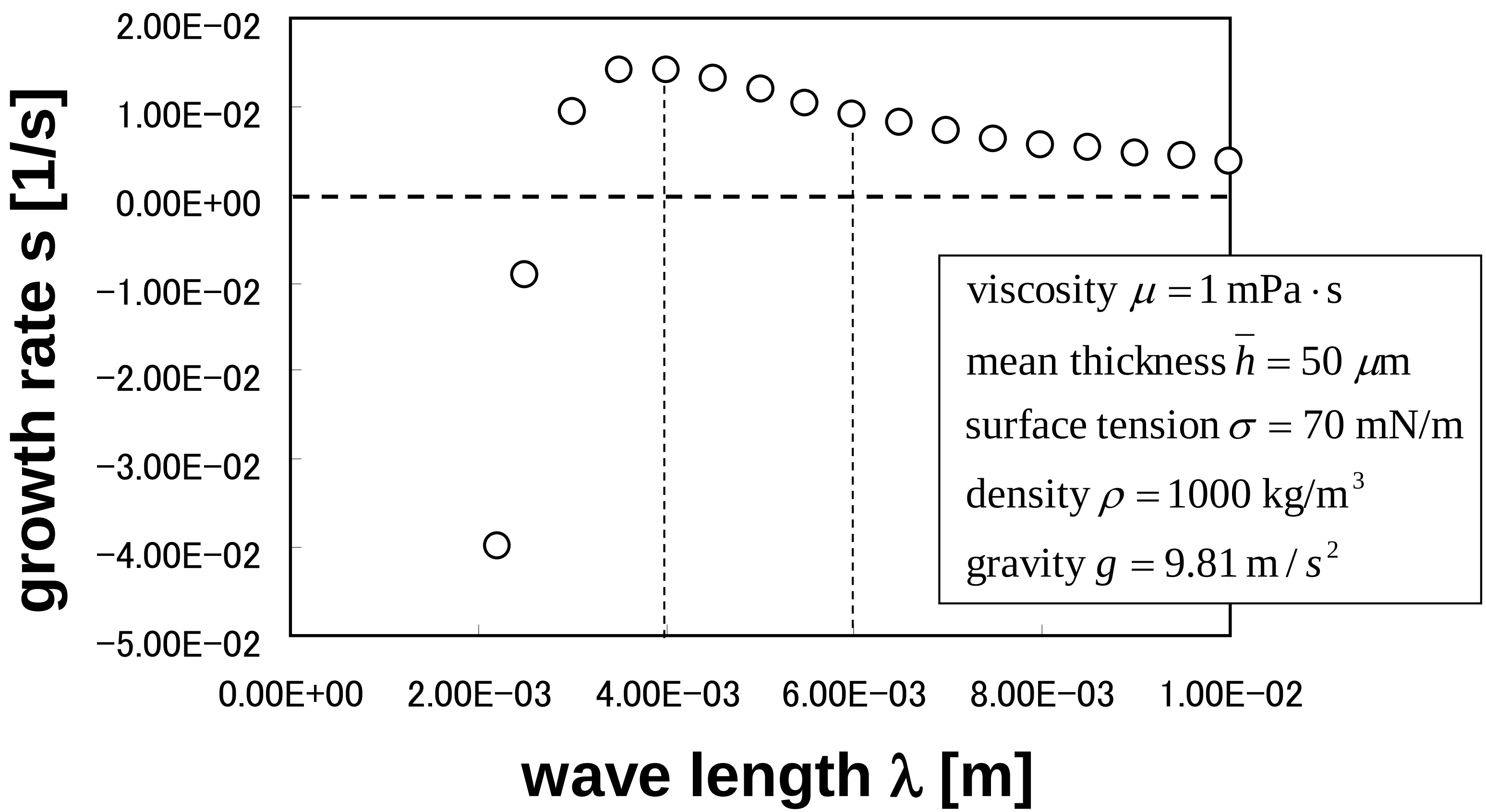
$$-\frac{\sigma}{\lambda^2}+\rho g < 0$$

$$\therefore \lambda < \sqrt{\frac{\sigma}{\rho g}}\ (\equiv \lambda_c)$$

単位チェック



LINEAR STABILITY ANALYSIS (5)



λ=2mm (凹凸は減衰：安定)
λ=4mm（凹凸が最も速く成長：不安定）
λ=6mm（凹凸は成長：不安定）

LINEAR STABILITY ANALYSIS (6)

$\lambda = \sqrt{\dfrac{\sigma}{\rho g}} (\equiv \lambda_c)$のとき、振幅$\varepsilon_0$の変動は減衰も増幅もしない（中立条件）

sが最大値となる波長λ_mは

$$\frac{ds}{d\lambda} = \frac{1}{3\mu}\bar{h}^3 \frac{d}{d\lambda}\left\{\left(-\frac{\sigma}{\lambda^2} + \rho g\right)\frac{1}{\lambda^2}\right\}$$

$$= \frac{1}{3\mu}\bar{h}^3 \left\{\frac{2\sigma}{\lambda^3}\frac{1}{\lambda^2} + \left(-\frac{\sigma}{\lambda^2} + \rho g\right)(-\frac{2}{\lambda^3})\right\}$$

$$= \frac{1}{3\mu}\bar{h}^3 \frac{2}{\lambda^3}\left\{\frac{2\sigma}{\lambda^2} - \rho g\right\} = 0$$の解だから

$$\frac{2\sigma}{\lambda_m{}^2} - \rho g = 0$$

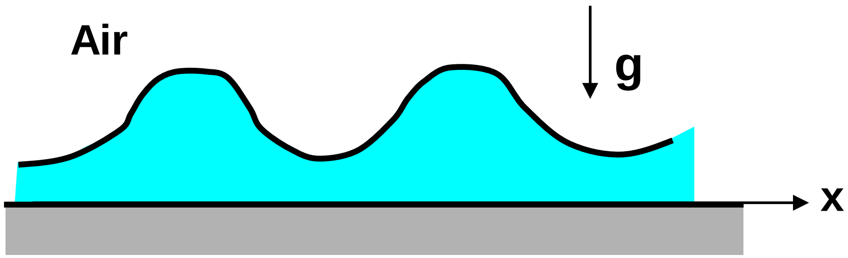
$$\lambda_m = \sqrt{\frac{2\sigma}{\rho g}} (= 2\lambda_c)$$

最大成長波長λ_mにおける成長速度s_mは

$$s_m = \frac{(\rho g)^2}{12\mu\sigma}\bar{h}^3$$

LINEAR STABILITY ANALYSIS (7) -leveling

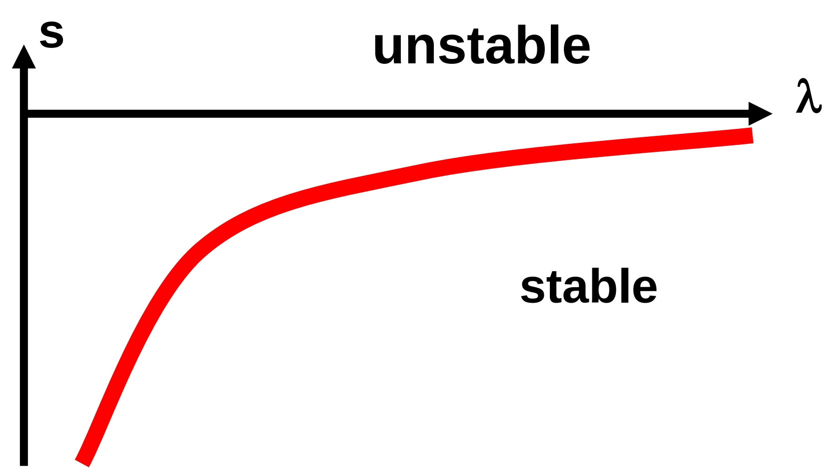
重量の方向を180°変更した場合



(4)の重力項の符号を変更すればよいから

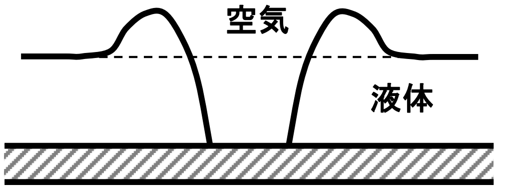
$$s = \frac{1}{3\mu}\bar{h}^3\left(-\frac{\sigma}{\lambda^2}-\rho g\right)\frac{1}{\lambda^2}$$

全てのλに対して$s<0$なので安定



Report 5 Linear stability analysis of thin film dewetting

氏名



Consider a thin liquid film on a solid surface. The intermolecular force, A/h^3, acts between the air-liquid and solid-liquid interface, where A is the Hamaker constant. The force is attractive and referred to as conjoining force when A is positive. Ass the film thins, the force increases and leads to spontaneous break-up of the liquid film. This phenomena is called “de-wetting” and often results in serous defects in industrial coating processes. Film profile equation is given by Eq. (1). The first and second terms on the right side of Eq. (1) represent the capillary force and conjoining force, respectively, where h(x) denotes the local film thickness, t is the time.

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{1}{3\mu}\left(\sigma h^3 \frac{d^3 h}{dx^3}\right) + \frac{A}{\mu}\frac{1}{h}\frac{dh}{dx}\right] \quad (1)$$

Use linear stability analysis to derive a critical condition of dewetting under constant surface tension, σ, and the viscosity, μ. The gravitational force is assumed to be negligible because the film is sufficiently thin.

Q1. Suppose an infinitely-small sinusoidal fluctuation in film thickness with the amplitude ε_0. The local thickness with the mean thickness $\bar{h}$ is expressed as Eq. (2) where λ represents the wavelength of the fluctuation. Derive Eq. (3) that expressed the growth rate of the fluctuation by substituting Eq. (2) into Eq. (1) and assuming $\varepsilon_0{}^2 \approx 0$ and $1/h \approx 1/\bar{h}$.

$$h = \bar{h} + \varepsilon_0 \exp(st)\sin(\frac{x}{\lambda}) \quad (2)$$

$$s = -\frac{\sigma}{3\mu}\frac{\bar{h}^3}{\lambda^4} + \frac{A}{\mu}\frac{1}{\bar{h}\lambda^2} \quad (3)$$

Q2. Derive the expression for the critical wavelength, λ_m , at which the growth rate shows a maximum.